

## Coordinate geometry II (midpoints and distances)

- Find the midpoint and length of the segment  $[AB]$ , where:
  - $A = (-1, 0), B = (3, 5)$ ;
  - $A = (0, 5), B = (3, -1)$ ;
  - $A = (2, -3), B = (0, 0)$ ;
  - $A = (1.5, 3.2), B = (0.7, 1.6)$ .
- Knowing that  $C$  is the midpoint of the segment  $[AB]$ , fill in the gaps or state with a reason that it is impossible. (In one part there is more than one possible solution—where?)
  - $A = (2, 3), B = (\dots, \dots), C = (5, -1)$ ;
  - $A = (1, \dots), B = (4, \dots), C = (2, 5)$ ;
  - $A = (0, \dots), B = (-2, 3), C = (\dots, -1)$ ;
  - $A = (3, 1), B = (\dots, 1), C = (-2, \dots)$ ;
  - $A = (\dots, -1), B = (-1, \dots), C = (1, 1)$ ;
  - $A = (\dots, \dots), B = (0, 0), C = (-1, 1)$ ;
- Let  $A = (1, 2), B = (3, 4), C = (5, 6)$  and  $A', B', C'$  be the midpoints of the segments  $[BC], [AC]$  and  $[AB]$  respectively. Show that the triangles  $\triangle ABC$  and  $\triangle A'B'C'$  are similar.
- Let  $A = (-1, -2), B = (3, 0), C = (4, 3), D = (-2, 0)$ .
  - Prove that the quadrangle  $ABCD$  is a trapezium.
  - Show that  $[AB] \perp [AD]$ .
  - Infer from (i) and (ii) that  $[AD] \perp [CD]$ .
  - Calculate the area of the trapezium  $ABCD$ .
  - $E$  is the midpoint of  $[AD]$  and  $F$  is the midpoint of  $[BC]$ . Determine the coordinates of  $E$  and  $F$ .
  - Find the equation of the line  $(EF)$ .
  - Show that  $(AB) \parallel (EF) \parallel (CD)$ .
- The triangle  $\triangle ABC$  has vertices  $A = (-1, -1), B = (5, 2)$  and  $C = (2, 5)$ .
  - Find the equation of the line  $(CD)$ , where  $D \in [AB]$  and  $(CD) \parallel (AB)$ .
  - Compute the coordinates of the point  $D$ .
  - Find the distances  $AB$  and  $CD$ .
  - What is the area of the triangle  $\triangle ABC$ ?
- If  $A = (4, 1)$  and  $B = (2, 2)$ , find the equation of the line  $l$  perpendicular to the segment  $[AB]$  and passing through its midpoint.
- Let  $A = (-1, 0), B = (3, 2), C = (2, 5)$ . The lines  $a, b, c$  pass through the vertices  $A, B, C$  and are perpendicular to the sides  $[BC], [AC]$  and  $[AB]$  of the triangle  $\triangle ABC$  respectively. Show that  $a, b, c$  all intersect at one point  $S$  and find its coordinates. Prove that  $AS = BS = CS$ .
- Given that  $A = (1, 2)$  and  $B = (4, -2)$ , calculate the coordinates of the point  $C$  such that the triangle  $\triangle ABC$  is isosceles. How many solutions does this problem have?

9. A *midline* is a line joining a vertex of a triangle with the midpoint of the opposite side. It can be proved that all three midlines in any triangle intersect at one point, called the *centroid* of the triangle. Check this fact for the triangle  $\triangle ABC$ , where  $A = (-1, 3)$ ,  $B = (0, 0)$ ,  $C = (1, 4)$ .
10. The sides of a triangle are included in the lines  $y = x$ ,  $y = -\frac{1}{4}x + 5$  and  $7x - 2y + 10 = 0$ . Calculate the area of this triangle.

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