

Exponential and trigonometric functions

1. Given are the following functions:

- (a) $f(x) = 2^x$;
- (b) $g(x) = 1 + (0.5)^x$;
- (c) $h(x) = (1.2)^{2x} - 8$;
- (d) $k(t) = 2^{x-1} + 3$;
- (e) $l(p) = 27 - 3^x$;
- (f) $x(s) = 500 + 20^s$;
- (g) $q(r) = -3 - 2^{2x+3}$.

For each of them:

- (i) sketch its graph,
- (ii) find its range,
- (iii) write down the equation of the asymptote and
- (iv) determine its value at 0.

2. During the chemical processing of some type of mineral, the amount M [kg] of the mineral at time t hours since the process started is given by

$$M(t) = M_0 2^{kt}, \quad t \geq 0,$$

where M_0 is the initial amount of the mineral present. We know that 128 kg of the mineral are reduced to 32 kg in the first six hours of the process.

- (a) Find the value of k and explain its sign.
- (b) Compute the quantity of the mineral that remains after 2 hrs of processing and round it to the nearest integer.
- (c) In what time is the mass of the mineral going to be equal to 12.7 kg?
- (d) Sketch the graph of M versus t .

3. Sketch the graphs of the following functions:

- (a) $y(x) = 2 \cos 4x - 3$;
- (b) $s(t) = 60 \sin 120x + 10$;
- (c) $x(u) = -5 \cos 0.25x + 8$.

4. Find a function $y = y(x)$ fitting the following data:

x	1	2	3	4	5	6	7	8	9	10
y	1.3	0.9	2.1	4.1	6.1	5.8	4.0	1.8	0.7	1.8
x	11	12	13	14	15	16	17	18	19	20
y	3.9	6.0	6.0	4.5	2.2	0.8	1.3	3.9	5.7	6.2

5. Solve the equation $3 \sin 20x = -4 + 1.5^x$.

6. Let $A = (6, 2)$, $B = (0, 3)$ and $C = (-1, 1)$.

- (a) Prove that the triangle $\triangle ABC$ is right-angled.
- (b) Write down the equation of the line l perpendicular to (AC) and passing through B .
- (c) Let D be the point of intersection of the lines (AC) and l . Compute the coordinates of D .
- (d) What are the distances AC and BD ?

- (e) Use your result from the previous part to calculate the area of $\triangle ABC$.
- (f) Find the mid-point B' of the side $[AC]$.
- (g) Show that the area of $\triangle A'BC$ is equal to the area of $\triangle ABA'$.

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