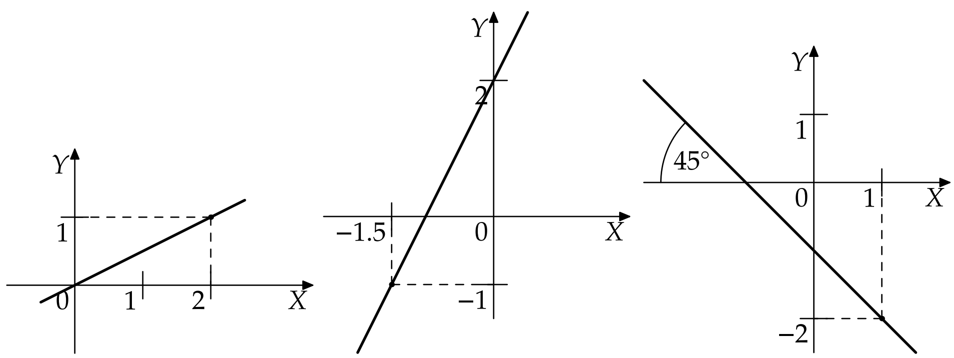
Equation of a line

1. Sketch the graphs of the following functions:

(a) $y = 2x + 3$;
(b) $y = -x + 1$;
(c) $y = 0.5x - 0.5$;
(d) $y = 5x - 8$;
(e) $y = -\frac{1}{3}x + \frac{2}{3}$;
(f) $y = -3$;
(g) $y = 200x - 300$;
(h) $y = 0.001x + 50$.

2. Find the equations of the following lines:



(a) (b) (c)

Represent them in the form $ax + by + c = 0$, where $a, b, c \in \mathbb{Z}$.

3. Write down the equation of the straight line passing through the points A and B in both the gradient-intercept form and in the form $ax + by + c = 0$ with a, b, c integer.

(a) $A = (0, 2), \quad B = (3, 0)$;
(b) $A = (-1, 3), \quad B = (2, 1)$;
(c) $A = (5, -4), \quad B = (-1, -4)$;
(d) $A = (-\frac{1}{2}, \frac{3}{4}), \quad B = (\frac{3}{4}, \frac{3}{2})$;
(e) $A = (-2.5, 0.5), \quad B = (1.5, 1)$;
(f) $A = (3, -2), \quad B = (3, 1)$.

4. Find the equation of the line passing through the point C and having the gradient equal to m. Write it down in the gradient-intercept form and in the form $ax + by + c = 0$ with $a, b, c \in \mathbb{Z}$.

(a) $C = (1, 2), \quad m = 1$;
(b) $C = (1, 2), \quad m = -1$;
(c) $C = (-1, 3), \quad m = 2$;
(d) $C = (0, 0), \quad m = 0$;
(e) $C = (3, -1), \quad m = -\frac{1}{2}$;
(f) $C = (3.81, 2.32), \quad m = 0$.

5. Find the equation of a straight line with Y-intercept n and passing through the point D. Represent them in the form $ax + by + c = 0$ for $a, b, c \in \mathbb{Z}$ and in the gradient-intercept form.

(a) $n = 3, \quad D = (1, 2);$
(b) $n = 0, \quad D = (5, 3);$
(c) $n = -\frac{1}{2}, \quad D = (2, -1);$
(d) $n = 3, \quad D = (-1, 3);$
(e) $n = 2, \quad D = (0, 5);$
(f) $n = -2.5, \quad D = (1, 1.5).$

6. Find the equation of a line with X-intercept a and Y-intercept b and write it down in gradient-intercept form and in the form $Ax + By + C = 0$ with A, B, C integer.

(a) $a = 2, \quad b = 4;$
(b) $a = -1, \quad b = 2;$
(c) $a = 0, \quad b = 1.5;$
(d) $a = 3, \quad b = \frac{1}{2};$
(e) $a = 1, \quad b = 0;$
(f) $a = b = 0.$

7. State with a reason of the two lines given are perpendicular or parallel.

(a) $k\colon y = 2x - 3, \quad l\colon y = 3x - 2;$
(b) $k\colon y = -x + 1, \quad l\colon y = x - 2;$
(c) $m\colon y = 3x - 3, \quad p\colon y = 3x + \frac{1}{3};$
(d) $p\colon y = 3x - 3, \quad q\colon y = \frac{1}{3}x + 2;$
(e) $a\colon 2x - y + 4 = 0, \quad b\colon -4x + 2y + 8 = 0;$
(f) $s\colon 2x + 3y + 4 = 0, \quad t\colon 6x - 4y - 2 = 0.$

8. Given a point P and a line k, find the equations of the lines: l, perpendicular to k and passing through P, and m, parallel to k and passing through P.

(a) $k\colon y = x + 2, \quad P = (3, 1);$
(b) $k\colon y = -2x + 3, \quad P = (0, 0);$
(c) $k\colon y = \frac{1}{3}x - \frac{2}{3}, \quad P = (11, 3);$
(d) $k\colon 2x - y + 1 = 0, \quad P = (1, 3);$
(e) $k\colon -x + 2 = 0, \quad P = (3, 1);$
(f) $k\colon y = 3, \quad P = (1, 0).$

9. Write down the following line equations in the form $ax + by + c = 0$, where $a, b, c \in \mathbb{Z}$, or state with a reason that it is impossible.

(a) $y = 2x - 1.5;$
(b) $y = \frac{1}{8}x + \frac{1}{4};$
(c) $y = -4x + 2.5;$
(d) $y = 5x - 25;$
(e) $y = 3;$
(f) $y = 3.3x + 2.1;$
(g) $y = 3x + \sqrt{2}.$

10. Write down the following line equations in the gradient-intercept form.

(a) $3x - 5y + 2 = 0$;

(b) $x\sqrt{2} + y\sqrt{3} + \sqrt{5} = 0$;

(c) $-\frac{1}{2}x + \frac{2}{3}y + \frac{3}{7} = 0$;

(d) $x + y = 0$.

11. Two lines, p and q, have equations: $p\colon\ y = (m-2)x + m$, $q\colon\ y = -mx + 2m^2$, where $m \in \mathbb{R}$ is a constant. For what values of m are they parallel or perpendicular?

12. Prove that a line with X-intercept a and Y-intercept b, where $a, b \neq 0$, has an equation of the form

$$\frac{x}{a} + \frac{y}{b} = 1.$$